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ABSTRACT

Logical inference tasks were presented to 112 police officers. Two logical forms were employed, converse and contrapositive. Three levels of activation were identified by a Q-Sort and incorporated into the design. A third factor was also considered, four styles of negative. Results of 24 items were analyzed by means of sign tests. Converse tasks were found to be easier than contrapositive tasks. Increased activation improved performance on the converse tasks but seemed to interfere with performance on the more difficult contrapositive tasks. Of special interest was the finding that contrapositive tasks with more negatives were easier than those with fewer negatives. (LS)

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THE RELATIONSHIP BETWEEN ACTIVATION AND LOGICAL INFERENCE PERFORMANCE

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THE RELATIONSHIP BETWEEN ACTIVATION AND LOGICAL INFERENCE PERFORMANCE

If the student is to study mathematics with appreciation, enjoyment, and success, he must be strengthened by knowledge of the necessary logical concepts. Mathematics educators agree that the concept of the implication is essential (Exner and Rosskopf, 1972, Secondary School Mathematics Curriculum Improvement Study, 1968; Van Engen, 1970; Comprehensive School Mathematics Program, 1971). The implication, with the valid logical deductions one can make from it--the implication with the invalid inferences one must be cautioned to avoid--is indeed one of the keys to mathematics.

In the present study, a group of police officers who went to college part-time were given logical inference tasks based on the implication. The tasks employed two logical forms, the converse and the contrapositive. The tasks were presented in verbal highly exciting police materials, routine police materials, as well as in abstract letter symbols. Thus the logical tasks varied in form and the content varied in level of stimulation.

METHOD

The Forms

To clarify the method of the study illustrations and definitions of the logical forms will be given.

The converse:

- Given
  Implication: "If a shot misfires, then a man needs time to aim again."
- Converse
  Implication: "Suppose a man needs time to aim again, then a shot misfired."

But, can one conclude that "A shot misfired"? Perhaps. But perhaps he sneezed before he had a chance to shoot or perhaps something else happened. There is no certainty. It is therefore impossible to determine whether or not the shot misfired. A student who accepts the converse, "Suppose a man needs time to aim again, then a shot misfired," is making an invalid deduction.

An implication is a sentence with two parts called the antecedent and the consequent. The "if" part is the antecedent; the "then" part is the consequent. To form the converse, one transforms the antecedent into the consequent, and vice versa.

The contrapositive:

- Given
  Implication: "If you scream, then I'll slash your throat."
- Contrapositive
  Implication: "Suppose the man did not slash your throat, then you did not scream."

> But, is it valid to conclude that "You did not scream"? We can try "You screamed." However, if you screamed, he would slash your throat and the man did not slash your throat. "You screamed" must be eliminated entirely. If "you screamed" is eliminated, then the only other possibility is "you did not scream."

To form a contrapositive, the antecedent is negated and becomes the new consequent. The consequent is negated and becomes the new antecedent. The original implication is valid if (and only if) the newly formed contrapositive statement is valid.

The Content

The logical tasks were both verbal and abstract. The verbal ones were embedded in paragraphs or vignettes. Two verbal tasks are included exactly as they appear in the instrument to illustrate levels of stimulation in the content. The first task requires the subject to respond to a converse implication in exciting content. In each vignette, the first question is a general one while the second refers to an implication embedded within the paragraph. It is this second question which is of interest in this study.

> Amidst screams of "Robbery!" I spotted two men dash out of a bank toward an empty parking lot. I shouted to them to drop their weapons and come along peaceably. But one stick-up man cocked his gun, pointed it at me and pulled the trigger. There was no explosion: that shot misfired. If a shot misfires, then a man needs time to aim again. I fired before he recovered and caught him in the shoulder.

a. In spite of the misfired shot, there was sufficient time to

(1) knock the robber out

(2) fire again

(3) neither (1) nor (2).

b. Suppose a man needs time to aim again, then

(1) a shot misfired

(2) a shot did not misfire

(3) it is impossible to determine whether or not a shot misfired.

The second task illustrates a contrapositive logical form embedded in a vignette with less exciting, more routine content. It is given below:

I was approached by a man who had a black wallet in his hand. He wanted it to be returned to its owner. It had some change, credit cards, and a safe deposit key in it. I delivered it to the desk officer of the precinct in which it was found. He telephoned the description to the Lost Property Unit to ascertain whether it had been reported lost or stolen. If I obtain lost property, then I deliver it to the desk officer.

a. Information about lost property is routed from

(1) police officer to precinct desk officer to Lost Property Unit

(2) police officer to Lost Property Unit to precinct desk officer

(3) neither (1) nor (2)

b. Suppose I do not deliver property to the desk officer, then

(1) I obtained lost property

(2) I did not obtain lost property

(3) It is impossible to determine whether or not I obtained lost property.

The next two illustrations are abstract. For the abstract converse given below, the correct answer is (3).

If statement p is accepted, then statement k follows.

Suppose statement k is accepted, then

(1) statement p follows

(2) the negation of statement p follows

(3) it is impossible to determine whether statement p or the negation of statement p follows.

Since no valid conclusion exists for a converse, it has been designated the invalid task.

For the following abstract contrapositive task, the correct answer is (2).

If statement v is accepted, then statement w follows.

Suppose the negation of statement w is accepted, then

(1) statement v follows

(2) the negation of statement v follows

(3) it is impossible to determine whether statement v or the negation of statement v follows.

Since it is always possible to make a logical deduction for a contrapositive,

it has been designated the valid task.

As can be seen from the illustrations of the converse and the contrapositive cited above, the ease with which the deductive argument is followed is not the same in each instance. This is due in part to the differences in the logical structures of the two forms. But, in addition, the content is surely a factor. The meaning of the words affects one's awareness of the structure of the argument and this, in turn, affects one's ability to follow the logical deduction. Psychologists and mathematics educators have been aware that the nature of the subject matter, or "content", as it has been called in the literature, has an effect on the performance of a logical task (Wilkins, 1928; Morgan and Morton, 1944; Feather, 1964; Carroll, 1970, Morgan, 1971).

One aspect of content that may make it possible to dimensionalize content more satisfactorily is activation. The individual's reaction to the logical tasks was presumably affected by the resolution of conflicts while he read and thought. During thinking the individual struggles with the variety of meanings that the words convey, decides on the sentences needed for the logical task, and performs the task. Materials that are novel or complex, or that have the power to induce uncertainty, have the qualities Berlyne describes as the collative determinants of activation. The verbal tasks were divided into two categories called More Activation and Less Activation. The third set consisted of Abstract tasks.

The Negatives

Recent research has suggested that the presence of negatives in a sentence may make its meaning more obscure. Thus, the effect of negatives on task difficulty was also investigated.

The original implication may be further complicated by the addition of negatives, i.e., the contrapositive of

If the negation of statement p is accepted, then statement q follows.

If the negation of statement q is accepted, then statement p follows.

Actually, the consequent "statement p follows" is the result of simplification of a double negative; it is not so that the negation of statement p follows. Therefore, a construction such as "statement p follows" can result from the simplification of a sentence that originally had two negatives. It seems likely that a phrase such as this, when it results from simplifying a double negative, would make the task more complex.

To study negation systematically, four styles of the assumed implication were used, for both converse and contrapositive tasks. They were:

	Symbolically:
1) If p is accepted, then q follows.	1) $p \rightarrow q$
2) If the negation of p is accepted, then q follows.	2) not $p \rightarrow q$
3) If p is accepted, then the negation of q follows.	3) $p \rightarrow$ not q
4) If the negation of p is accepted, then the negation of q follows.	4) not $p \rightarrow$ not q

Converse and contrapositive tasks were written in each style, as presented in Table 1. The number of negatives in each argument is indicated.

TABLE 1

Negation in Tasks

Negative Level	Style	Converse	Negs.	Contrapositive	Negs.
Fewer	p → q	q → ?	0	not q → ?	1
Fewer	not p → q	q → ?	1	not q → ?	2
More	p → not q	not q → ?	2	not not q → ? or q → ?	3
More	not p → not q	not q → ?	3	not not q → ? or q → ?	4

The Design

The design of the present study included the creation of verbal tasks and the selection of paragraphs in which the logical inference tasks were embedded. The final paragraphs had come from a pool of thirty-five paragraphs which had been presented to thirty-four police officers. They had indicated the level of excitement of each paragraph according to a seven-point Q-Sort. A Q-Sort is a rank ordering process in which items are arranged along a continuum which ranges from the lowest degree of a particular characteristic to the highest. The results of the Q-Sort provided two sets of paragraphs called Less Activation and More Activation. It was thought that the portrayal of police in a positive or negative light might effect performance on logical tasks. A second Q-Sort was administered to separate paragraphs according to positive or negative police image. Thus there were eight More Activation paragraphs and eight Less Activation paragraphs. Each set of eight was balanced for police image. Eight additional tasks in letter symbols were written. Thus, there were twenty-four questions containing three levels of activation, two forms, and four styles of negation.

The Sample

The subjects were a group of 112 police officers who went to college part-time. They were taking or had taken a basic course in college mathematics which contains a unit on logic (group 1). For the second semester of mathematics, students with interest in the humanities enrolled in a course with an historical approach (group 2), while the remainder studied applications of mathematics (group 3). This allowed for a comparison of groups.

The Hypotheses

The purpose of the study was to investigate the effects of the three activation levels of content, the two logical inference forms , and the two degrees of negatives on logical task performance of the sample.

In accordance with this purpose, the following hypotheses were investigated:

H-1: For both the converse and the contrapositive, students do not perform equally well on More Activation, Less Activation, and Abstract tasks.

H-2: Students perform better on valid arguments as represented by the contrapositive, than on invalid arguments, as represented by the converse.

H-3: For both the converse and the contrapositive, students perform better where there are fewer negatives than where there are more negatives.

RESULTS

The students' answers to the twenty-four questions provided data to investigate three hypotheses. The first hypothesis was:

H-1: For both the converse and the contrapositive, students do not perform equally well on More Activation, Less Activation and Abstract tasks.

The hypothesis was supported in sixteen out of eighteen instances. Only Group 2 did not find More Activation tasks significantly easier then Less Activation tasks.

For the converse:

a) More Activation tasks are significantly easier at the 5 percent level than Less Activation tasks, for two out of three groups.

b) Abstract tasks are significantly easier at the 5 percent level than More Activation tasks, for all three groups.

c) Abstract tasks are significantly easier at the 5 percent level than Less Activation tasks, for all three groups.

For the contrapositive:

a) Less Activation tasks are significantly easier at the 5 percent level than More Activation tasks, for two out of three groups.

b) More Activation tasks are significantly easier at the 5 percent level than Abstract tasks, for all three groups.

c) Less Activation tasks are significantly easier at the 5 percent level than Abstract tasks, for all three groups.

The sign test was the primary measure used in testing the hypothesis. The proportion of instances in which subjects scored higher on one type of task in comparison to another was determined. The data is summarized in Tables 2 and 3.

The second hypothesis was:

H-2: Students perform better on valid arguments as represented by the contrapositive than on invalid arguments as represented by the converse.

The hypothesis was rejected, with an interesting reversal occurring in one of the three groups. The invalid converse tasks are significantly easier at the 5 percent level than valid contrapositive tasks for Group 3. Z scores for each of the three differences were again calculated and summarized in Table 4.

Means and standard deviations were calculated and recorded in Table 5. For each group, the mean on invalid converse tasks was decidedly higher than for valid contrapositive tasks.

The third hypothesis was:

H-3: For both the converse and the contrapositive, students perform better where there are fewer negatives than where there are more negatives.

The hypothesis was rejected with an unexpected reversal occurring on the contrapositive.

For the converse:

Tasks with fewer negatives are not significantly different from tasks with more negatives for each of the three groups.

TABLE 2

Sign Test for Comparing Converse Tasks Which Differ in Activation Level

		CONVERSES					
Group		Activation Differences					
		More - Less		Less - Abstract		Abstract - More	
i	N	z_i	$P(\|z\| > \|z_i\|)$	z_i	$P(\|z\| > \|z_i\|)$	z_i	$P(\|z\| > \|z_i\|)$
1	44	3.2	.00010	-4.8	.00006	2.8	.005
2	43	1.7	.089	-3.1	.00194	3.3	.00097
3	25	2.1	.036	-3.3	.00097	2.5	.012
Total	112	4.6	.00006[a]	-6.5	.00006[a]	5.0	.00006[a]

[a] The exact value is somewhat less than .00006

$$z_i = \frac{P - 1/2}{\sqrt{1/4\ n}}$$

TABLE 3

Sign Test for Comparing Contrapositive Tasks
which Differ in Activation Level

Group	Contrapositives					
	Activation Differences					
	More - Less		Less - Abstract		Abstract - More	
i	z_i	$P(\|z\| > \|z_i\|)$	z_i	$P(\|z\| > \|z_i\|)$	z_i	$P(\|z\| > \|z_i\|)$
1	-4.2	.00006[a]	4.1	.00006	-2.3	.021
2	-1.6	.110	5.1	.00006[a]	-4.6	.00006[a]
3	-4.2	.00006[a]	3.7	.00022	-2.3	.021
Total	-5.7	.00006[a]	7.5	.00006[a]	-5.4	.00006[a]

[a]The exact value is somewhat less than .00006.

$$z_i = \frac{p - \frac{1}{2}}{\sqrt{\frac{1}{4} n}}$$

TABLE 4

Sign Test for Comparing Valid and Invalid Tasks

Group i	N	s[a]	f[b]	$p = \frac{s}{s + f}$	$\frac{p - \frac{1}{2}}{\sqrt{n \frac{1}{4}}}$	$P(\lvert z \rvert > \lvert z_i \rvert)$
1	44	20	22	.4761	-.3	.764
2	43	18	22	.45	-.6	.549
3	25	6	17	.2608	-2.3	.021

[a]s or success is the number of positive differences between scores on Valid and Invalid tasks.

[b]f or failure is the number of negative differences between scores on Valid and Invalid tasks.

TABLE 5

Mean and Standard Deviation Scores of Each Group for Valid and Invalid Tasks

Category	Group 1 N = 44 Mean	S.D.	Group 2 N = 43 Mean	S.D.	Group 3 N = 25 Mean	S.D.
Valid	4.8	3.0	4.9	2.6	3.8	2.5
Invalid	6.3	3.0	6.1	2.8	7.2	3.0

For the contrapositive:

> Tasks with more negatives are significantly easier at the 5 percent level than tasks with fewer negatives for all three groups.

The data is summarized in Table 6.

TABLE 6

Sign Test for Comparing Tasks which Differ in Degree of Negation

Group i	N	Converse Tasks[a] z_i[d]	Converse Tasks[a] $P(\|z\|>\|z_i\|)$	Contrapositive Tasks[b] z_i[d]	Contrapositive Tasks[b] $P(\|z\|>\|z_i\|)$
1	44	-.4	.689	-3.9	.00010
2	43	1.3	.194	-3.3	.00097
3	25	-.3	.764	-3.4	.00067
Total	112	.5	.617	-6.1	.00006[c]

[a]Converse tasks with 0 or 1 negatives are being compared with converse tasks that have 2 or 3 negatives.

[b]Contrapositive tasks with 1 or 2 negatives are being compared with contrapositive tasks with 3 or 4 negatives.

[c]The exact value is somewhat less than .00006.

[d]$z_i = \frac{p - \frac{1}{2}}{\sqrt{\frac{1}{4} n}}$

CONCLUSIONS

The converse tasks were easier than the contrapositive tasks, significantly so for group 3. For the simpler form, the converse, the addition of exciting content in the More Activation tasks improved performance over that of the Less Activation tasks. One is led to believe that interesting materials can stimulate in an effective way when the task is sufficiently easy.

On the other hand, with the more difficult form, the contrapositive, the increase in excitement in the More Activation materials may have created too much activation, more than was desirable. The combination of difficult tasks with exciting content in the More Activation contrapositive may cause conflict. The subject can either think about possible outcomes of the incident related in the vignette or think about the logical task itself. If he chooses not to accept the logical task, his performance declines. "It may be that a strong emotional involvement with particular material is, in part, responsible for the difficulty which many unsophisticated subjects experience in distinguishing between drawing a conclusion that is logically valid and one that is believed to be correct" [Henle, 1962, p. 374].

It is fascinating to note that when students make deductions from two logical structures, stimulating language -- even on the printed page -- sometimes makes the logical task easier. For the easier structure, performance increases.

Does this have relevance for the learning of other conceptual structures? Can we make finer distinctions in cognitive content to achieve optimum performance by adjusting the affective dimension?

Only for the contrapositive was the number of negatives in the implication related to success in task performance. Strangely enough, contrary to the prediction, contrapositive tasks with more negatives were easier than contrapositives with fewer. Results were significant for all groups. The conclusions are in accord with O'Brien's results (1972) on the contrapositive form. This represents a change from earlier research results that the addition of negatives to the standard form of a principle of logic made the task more difficult (Hill, 1960; O'Brien 1971).

Further scrutiny of similarities between the more difficult contrapositives reveals that it may not be the number of negatives that causes difficulty, but rather the location of negatives. In both of these more difficult styles, the antecedent of the contrapositive contains a negative. This is not the case in the two easier contrapositive styles. For these contrapositives, the antecedent clause has no negatives. Could it be that an important determinant of difficulty for the contrapositive form is the presence or absence of negation in this antecedent clause?

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